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NUMERICAL AND COMPUTATIONAL ANALYSIS REGARDING BUCKLING KNOCKDOWN FACTOR OF COMPOSITE CYLINDERS

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Abstract. *Since the publication of the guideline NASA SP-8007 for buckling of thin walled shells their knockdown factor has been largely referenced as the empirical parameter that takes into account the differences between numerical and experimental critical buckling load on thin walled cylinders. These differences are mainly attributed to imperfections of the real structure due to its manufacture process or real load case conditions. Although the NASA SP-8007 guideline is not adequate for composite laminate shells due to anisotropy coupling effects researchers and designers take its knockdown factor as a reference value for comparison with actual methods in development. This research approaches a numerical analysis of the knockdown factor based on a data set of composite cylinders subject to buckling behavior taken from the literature comparing it with finite element simulation and an analytical method based on the Sander's theory. Presented conclusions over the obtained results regarding composite cylindrical shells knockdown factor shows that with both methods is possible to determine less conservative factors than those from NASA SP-8007.*

1. INTRODUCTION

Currently the buckling analysis of thin walled composite cylinders is mainly done by non-linear finite element analysis with real geometric imperfections and load case. However, in an early stage of design this data do not exist and an alternative is to calculate the linear buckling load using linear bifurcation analysis and to correct the obtained result by a knockdown factor (KDF) [1].

Since the publication of the guideline NASA SP-8007 for buckling of thin walled shells their knockdown factor has been largely referenced as the empirical parameter that takes into account the differences between numerical and experimental critical buckling load on thin walled cylinders [2]. These differences are mainly attributed to imperfections of the real structure due to its manufacture process or real load case conditions. Although the NASA SP-8007 guideline is not adequate for composite laminate shells due to anisotropy coupling effects researchers and designers take its knockdown factor as a reference value for comparison with actual methods in development.

Different approaches has been developed in the last years as for example deterministic methods like the “Single Perturbation Load” (SPL) and probabilistic methods like Takano’s Statistical Approach. In the SPLA method a single perturbation load simulates the effect of the unknown imperfections for defining the knockdown factor as the lower bound of the structural buckling caused by it. This method requires a lot of numerical and laboratorial experiments for validation of the load magnitude. The Takano’s approach is based on a large experimental data set for the statistical knockdown factor determination and their probabilistic nature depends on the range of experimental data. Both of the cited methods are time consuming, largely dependent on experimental data and can result in over or under estimates of the knockdown factor if not well estimated [1]. A fast design approach for the early stage of buckling analysis is still required.

This research approaches a numerical analysis of the knockdown factor based on a data set of composite cylinders subject to buckling behavior taken from the literature comparing it with finite element simulation and an analytical method based on the Sander's theory presenting conclusions over the obtained results regarding composite cylindrical shells knockdown factors.

2. KNOCKDONW FACTOR COMPARISON

The numerical comparison of knockdown factors for composite cylinders will be made based on an experimental data set collected from literature. These data will be compared with results of a finite element linear bifurcation analysis, analytical methods based on the NASA SP-8007 and other based on Sander’s non-linear equations for thin elastic shells, and with recommended knockdown factor from NASA SP-8007. Figure 1 adapted from [3] illustrates the knockdown factor concept, representing the difference between the compressive axial buckling load of an ideally perfect structure, N_{per}, and that of the real structure, N_{imp}. Knockdown factor (KDF) is then calculated by Eq. (1).

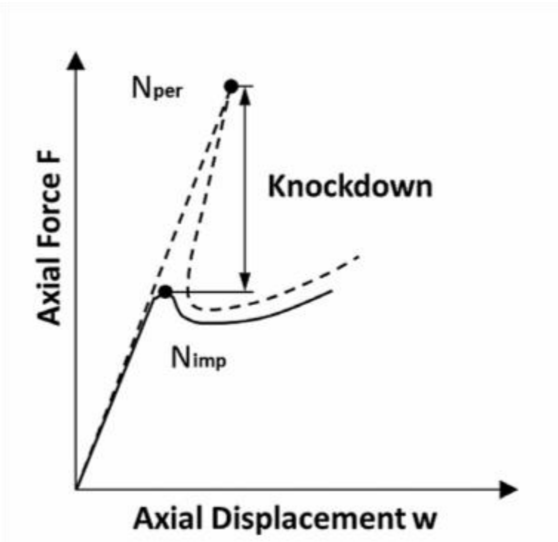


Figure 1 – Buckling loads defining the knockdown factor concept.

$$KDF = \frac{P_{imp}}{P_{per}} \tag{1}$$

where P_{imp} is the real buckling load of an experimental test and P_{per} is the buckling load of an ideally perfect structure obtained by numerical methods.

2.1. NASA SP-8007

The NASA SP-8007 [4] indicates an analytical method for the calculation of buckling load of orthotropic cylinders under axial compression based on Jones deduction for orthotropic thin walled cylinders and also a direct calculation of the recommended KDF for such cylinders showed here as Eq. (2) and here denominated KDF_{NASA}.

$$KDF_{NASA} = 1 - 0{,}902(1 - e^{-\phi}) \tag{2}$$

$$\phi = \frac{1}{16}\sqrt{\frac{R}{3{,}4689\sqrt[4]{\dfrac{D_{11}D_{22}}{A_{11}A_{22}}}}} \tag{3}$$

where R is the mid-surface cylinder radius, A_{11}, A_{22}, D_{11} and D_{22} are the terms of the composite laminate ABD stiffness matrix.

Here it's worth to mention that Eq. (2) is originally a curve fitting based on experimental data set of isotropic cylinders tests and that Eq. (3) is a modified factor for orthotropic material.

2.2. Analytical method based on Sanders

The analytical method based on Jones orthotropic assumptions are not adequate for a composite laminate analysis considering its full anisotropy and for this reason another analytical method will be evaluated based on Sanders nonlinear shell theory following the deductions presented by Nemeth in [5] implemented in a numerical routine created with the Matlab software. Following the work of Nemeth buckling mode shapes can be represented by a displacement field that includes a skewedness parameter and this represents an enhancement over the classic orthotropic displacement field allowing for an angular orientation. The displacement field with the skewedness parameter is calculated with Eq. (4), Eq. (5) and Eq. (6).

$$u = \bar{u}\,cos(\frac{m\pi x}{L})\,cos(\frac{n}{R}(y - \tau x)) \tag{4}$$

$$v = \bar{v}\,sin(\frac{m\pi x}{L})\,sin(\frac{n}{R}(y - \tau x)) \tag{5}$$

$$w = \overline{w}\, sin(\frac{m\pi x}{L})\, cos(\frac{n}{R}(y - \tau x)) \quad (6)$$

where u,v and w are the mid-surface displacements on the axial, circumferential and radial directions $\overline{u}$, $\overline{v}$ and $\overline{w}$ are its amplitudes, τ is the skewedness parameter, m and n are the number of half waves of the buckling pattern on axial and circumferential directions, x and y are axial and circumferential coordinates of the cylinder.

2.3. Finite element analysis (FEA)

A computational linear bifurcation analysis was performed with the finite element software ABAQUS for a perfect cylindrical structure. The cylindrical structure was modeled with S8R5 shell elements and the mesh size was defined after a mesh sensitivity analysis. Axial compressive force was applied on extremities nodes and the boundary conditions adopted was clamped with axial degree of freedom on both sides.

Buckling mode extraction was performed by the subspace eigensolver and the results were inspected against spurious buckling modes. Compared to a nonlinear analysis this method is very time saving.

2.4. Experimental data set

A reliable experimental data set was selected from literature. This selection is a critical issue in this work in face of the deleterious effect of prototypes manufactures imperfections and imprecision on test results. Table 1 presents the material properties adopted in each case study and Table 2 presents the dimensional and stacking sequence information.

Table 1 – Unidirectional material properties of reference data set.

Reference	**E_1 [MPa]**	**E_2 [MPa]**	**G_{12} [MPa]**	**ν_{12} [*]**
[6]	125774	10030	5555	0,27
[7]	150000	9080	5290	0,32
[8]	122500	11000	3250	0,27
[9]	156900	10400	5260	0,28

Table 2 – Dimensional and stacking data of cylinders.

Reference	**Stacking**	**L [mm]**	**R [mm]**	**t [mm]**	**L/R [*]**	**R/t [*]**
	[±24/±41]	500	250	0,5	2	500
[6]	[±41/±24]	500	250	0,5	2	500
	[24/±41/-24]	500	250	0,5	2	500
[7]	$[\pm45]_S$	520	250	0,5	2	500
[8]	$[90/\pm30]_S$	215	115	0,81	2	142
[9]	[±24/±41]	500	250	0,5	2	500

2.5. Influences on buckling load estimation

As already mentioned geometrical and load imperfections are considered the principal causes of discrepancies between a real and an ideal structure buckling behavior and they are the main focus of researchers that pursue an exact result on buckling calculation. Also was mentioned that these information are not available in the early stage of structural analysis.

A surprising fact is that less attention is devoted for the real material properties of the actual structure or its nonlinear behavior. In general the composite material is characterized by standard tests coupons or specimens taken from the prototype for determination of unidirectional ply properties defining its Poisson ratio, tensile and shear modulus on the principal directions. Is a know fact that polymeric composite laminates have different modulus on compressive and tensile directions [9, 10] and also that buckling is mainly governed by compressive stresses on the structure. Another issue is that usually the employed constitutive material models do not allow for a multiple elastic modulus analysis. By these reasons the adopted elastic modulus values must be carefully evaluated for a buckling analysis.

From reference [9] we observe the difference between tensile and compression modulus values for a carbon/epoxy composite which are presented in Table 3. The mean value of 0,89 (89%) is the same of the mean value of several carbon fiber composites presented in reference [10] and so this ratio may be considered for numerical evaluation of longitudinal E_1 modulus. The transversal E_2 modulus ratio will be adopted as unit due to its low value compared with E_1 modulus.

Table 3 – Comparison between tensile and compression unidirectional modulus.

Reference	**E_{1t} [MPa]**	**E_{1c} [MPa]**	**E_{2t} [MPa]**	**E_{2c} [MPa]**	**E_{1c}/ E_{1t} ***	**E_{2c}/ E_{2t} ***
[9]	164100	142500	8700	9700	0,87	1,12
	175300	157400	8600	10100	0,90	1,17
	164000	150000	*	*	0,91	*
	171500	150200	8900	9400	0,87	1,06
				Mean value	0,89	1,12

3. RESULTS

Results of computational and numerical analysis are presented on the preceding tables. For each method a respective KDF was calculated. Tables 4 to 7 results were obtained using the material data from Table 1.

Table 4 – Results for case study on reference [6].

Stacking	**N_{EXP} [kN]**	**N_{JONES} [kN]**	**KDF_J [*]**	**$N_{SANDERS}$ [kN]**	**KDF_S [*]**	**N_{FEA} [kN]**	**KDF_F [*]**	**KDF_{NASA} [*]**
[±24/±41]	21,80	28,29	0,77	25,60	0,85	26,68	0,82	0,32
[±24/±41]	21,90	28,29	0,77	25,60	0,86	26,68	0,82	0,32
[±41/±24]	15,70	19,52	0,80	19,52	0,80	17,61	0,89	0,32

[24/±41/-24]	15,70	29,12	0,54	24,15	0,65	23,34	0,67	0,32
[24/±41/-24]	16,70	29,12	0,57	24,15	0,69	23,34	0,72	0,32

Table 5 – Results for case study on reference [7].

Stacking	**N_{EXP} [kN]**	**N_{JONES} [kN]**	**KDF_J [*]**	**$N_{SANDERS}$ [kN]**	**KDF_S [*]**	**N_{FEA} [kN]**	**KDF_F [*]**	**KDF_{NASA} [*]**
$[\pm45]_S$	15,35	26,94	0,57	26,92	0,57	23,28	0,66	0,32
$[\pm45]_S$	14,33	26,94	0,53	26,92	0,53	23,28	0,62	0,32
$[\pm45]_S$	14,41	26,94	0,53	26,92	0,54	23,28	0,62	0,32
$[\pm45]_S$	13,01	26,94	0,48	26,92	0,48	23,28	0,56	0,32
$[\pm45]_S$	12,75	26,94	0,47	26,92	0,47	23,28	0,55	0,32
$[\pm45]_S$	15,79	26,94	0,59	26,92	0,59	23,28	0,68	0,32

Table 6 – Results for case study on reference [8].

Stacking	**N_{EXP} [kN]**	**N_{JONES} [kN]**	**KDF_J [*]**	**$N_{SANDERS}$ [kN]**	**KDF_S [*]**	**N_{FEA} [kN]**	**KDF_F [*]**	**KDF_{NASA} [*]**
$[90/\pm30]_S$	60,20	88,80	0,68	74,26	0,81	73,80	0,82	0,53
$[90/\pm30]_S$	57,40	88,80	0,65	74,26	0,77	73,80	0,78	0,53
$[90/\pm30]_S$	62,10	88,80	0,70	74,26	0,84	73,80	0,84	0,53
$[90/\pm30]_S$	61,90	88,80	0,70	74,26	0,83	73,80	0,84	0,53
$[90/\pm30]_S$	60,40	88,80	0,68	74,26	0,81	73,80	0,82	0,53
$[90/\pm30]_S$	58,80	88,80	0,66	74,26	0,79	73,80	0,80	0,53
$[90/\pm30]_S$	61,70	88,80	0,69	74,26	0,83	73,80	0,84	0,53
$[90/\pm30]_S$	60,50	88,80	0,68	74,26	0,81	73,80	0,82	0,53
$[90/\pm30]_S$	56,20	88,80	0,63	74,26	0,76	73,80	0,76	0,53
$[90/\pm30]_S$	57,70	88,80	0,65	74,26	0,78	73,80	0,78	0,53
$[90/\pm30]_S$	55,40	88,80	0,62	74,26	0,75	73,80	0,75	0,53
$[90/\pm30]_S$	59,30	88,80	0,67	74,26	0,80	73,80	0,80	0,53

Table 7 – Results for case study on reference [9].

Stacking	**N_{EXP} [kN]**	**N_{JONES} [kN]**	**KDF_J [*]**	**$N_{SANDERS}$ [kN]**	**KDF_S [*]**	**N_{FEA} [kN]**	**KDF_F [*]**	**KDF_{NASA} [*]**
[±24/±41]	25,38	30,85	0,82	27,78	0,91	28,92	0,91	0,32
[±24/±41]	25,64	30,85	0,83	27,78	0,92	28,92	0,92	0,32

From the presented results we observe that KDF_{NASA} and the critical buckling load from Jones deduction for orthotropic cylinders were the worst.

As mentioned in section 2.5, for evaluation purposes the difference of 89 % on modulus E1 changing from tensile to compression direction was considered and it was implemented on the finite element analysis maintaining all the other properties unchanged. Results presented on Tables 8 to 11 were obtained.

Table 8 – Results for case study on reference [6] with 89 % E_1.

Stacking	N_{EXP} [kN]	N_{FEA} [kN]	$KDF_{F0,89}$ [*]
[±24/±41]	21,80	25,48	0,86
[±24/±41]	21,90	25,48	0,86
[±41/±24]	15,70	15,87	0,99
[24/±41/-24]	15,70	22,17	0,71
[24/±41/-24]	16,70	22,17	0,75

Table 9 – Results for case study on reference [7] with 89 % E_1.

Stacking	N_{EXP} [kN]	N_{FEA} [kN]	$KDF_{F0,89}$ [*]
$[\pm45]_S$	15,35	22,20	0,69
$[\pm45]_S$	14,33	22,20	0,64
$[\pm45]_S$	14,41	22,20	0,65
$[\pm45]_S$	13,01	22,20	0,59
$[\pm45]_S$	12,75	22,20	0,57
$[\pm45]_S$	15,79	22,20	0,71

Table 10 – Results for case study on reference [8] with 89 % E_1.

Stacking	N_{EXP} [kN]	N_{FEA} [kN]	$KDF_{F0,89}$ [*]
$[90/\pm30]_S$	60,20	68,20	0,88
$[90/\pm30]_S$	57,40	68,20	0,84
$[90/\pm30]_S$	62,10	68,20	0,91
$[90/\pm30]_S$	61,90	68,20	0,91
$[90/\pm30]_S$	60,40	68,20	0,89
$[90/\pm30]_S$	58,80	68,20	0,86
$[90/\pm30]_S$	61,70	68,20	0,90
$[90/\pm30]_S$	60,50	68,20	0,89
$[90/\pm30]_S$	56,20	68,20	0,82
$[90/\pm30]_S$	57,70	68,20	0,85
$[90/\pm30]_S$	55,40	68,20	0,81
$[90/\pm30]_S$	59,30	68,20	0,87

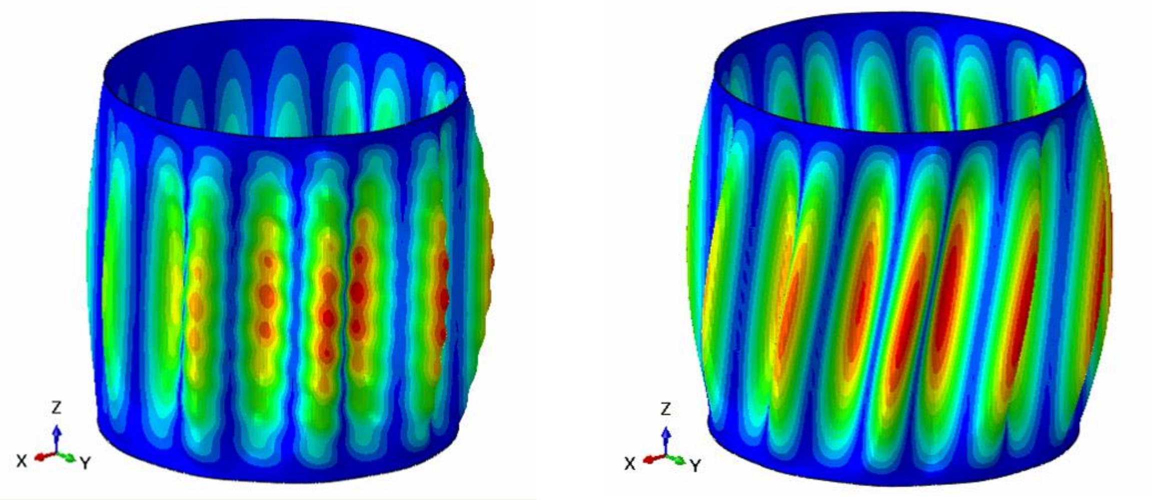
Table 11 – Results for case study on reference [9] with 89 % E_1.

Stacking	**N_{EXP} [kN]**	**N_{FEA} [kN]**	**$KDF_{F0,89}$ [*]**
[±24/±41]	25,38	27,65	0,92
[±24/±41]	25,64	27,65	0,93

Table 12 summarizes the obtained results. From these results is possible to observe how conservative is the KDF_{NASA} compared to those obtained by the implemented analytical method, KDF_S, or by the linear bifurcation analysis performed on the finite element software, KDF_F. Another observation is that with the 89% E_1 modulus consideration accounting for the tensile compression modulus differences the obtained result, $KDF_{F0,89}$, is more close related to the experimental buckling load result showing that this material behavior if correlated with test results. While from the NASA SP-8007 guideline the KDF results lay on the range of 0,32 to 0,53, all of the methods of the present study has obtained KDF values from 0,47 to 0,85 and from 0,55 to 0,89. Figure 2 shows the obtained critical buckling mode from finite element analysis for the stacking [24/±41/-24] with E_1 and 0,89%E_1 elastic modulus.

Table 12 – Minimum KDF obtained by method of analysis.

Stacking Reference	**[±24/±41] [6]**	**[±41/±24] [6]**	**[24/±41/-24] [6]**	**$[\pm 45]_S$ [7]**	**$[90/\pm 30]_S$ [8]**	**[±24/±41] [9]**
KDF_{NASA}	0,32	0,32	0,32	0,32	0,53	0,32
KDF_J	0,77	0,80	0,54	0,47	0,62	0,77
KDF_S	0,85	0,80	0,65	0,47	0,75	0,85
KDF_F	0,82	0,89	0,67	0,55	0,75	0,82
$KDF_{F0,89}$	0,86	0,79	0,71	0,57	0,81	0,86



(a) (b)

Figure 2 – Critical buckling modes obtained from FEA for [24/±41/-24] (a) with E1; (b) with 89% E1.

4. CONCLUSIONS

Based on the selected experimental data set results taken from the literature this study was able to demonstrated that the NASA SP-8007 guideline are not adequate for composite cylinders concerning its knockdown factor suggestions. It was possible to make a clear appointment on the conservative aspect of the NASA SP-8007 knockdown factor. This is a consequence of the anisotropy behavior of the composite laminate that cannot be estimated by similarity with isotropic material structural data due to the flexural-extensional coupling effect of the laminate. Also it become clear that the Jones deduction for orthotropic cylinders should not be used for composite laminate cylinders due to the fact that this method don't considers the full anisotropy of the laminate and imposes an orthotropic displacement field. Better results were obtained with the Sanders deduction presented by Nemeth and with the finite element linear bifurcation analysis.

Definition of reliable knockdown factors for composites is still a work in progress on the scientific field. Promising approaches are been developed by researchers over the world. For further investigations nonlinearities and real imperfections features must be implemented on a nonlinear finite element analysis to simulate pre and post buckling behaviors.

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